Math 155 Review from Calculus I
Revisit Integration and Differentiation

Name____________

Section 4.5 *Integration by Substitution*

THEOREM 4.12 Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C.$$

If $u = g(x)$, then $du = g'(x)\,dx$ and

$$\int f(u)\,du = F(u) + C.$$

Guidelines for Making a Change of Variables

1. Choose a substitution $u = g(x)$. Usually, it is best to choose the *inner* part of a composite function, such as a quantity raised to a power.
2. Compute $du = g'(x)\,dx$.
3. Rewrite the integral in terms of the variable u.
4. Find the resulting integral in terms of u.
5. Replace u by $g(x)$ to obtain an antiderivative in terms of x.
6. Check your answer by differentiating.

Ex.1 Integrate: $\displaystyle\int t^3\sqrt{t^4+5}\,dt$

Ex.2 Solve: $\dfrac{dy}{dx} = \dfrac{10x^2}{\sqrt{1+x^3}}$

Ex.3 Integrate: $\int \sec^2(x)\sqrt{\tan(x)}dx$

Change of Variables

Ex.4 Integrate: $\int x\sqrt{2x+1}\,dx$

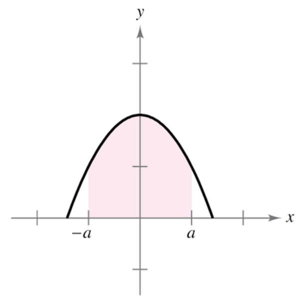
Even & Odd Functions

THEOREM 4.15 Integration of Even and Odd Functions

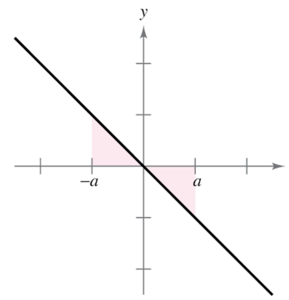
Let f be integrable on the closed interval $[-a, a]$.

1. If f is an *even* function, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

2. If f is an *odd* function, then $\int_{-a}^{a} f(x)\,dx = 0$.



Even function



Odd function

Ex.5 Integrate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin(x)}{\sqrt{1+\cos(x)}}dx$

Ex.6 Integrate: $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2(x)\cos(x)dx$

Ex.7 Integrate: $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right)d\theta$

Ex.8 Integrate: $\int \frac{x^2-1}{\sqrt{2x-1}}dx$

Section 5.4 *Logarithmic, Exponential, & other Transcendental Functions*

Ex.1 Find $\dfrac{dy}{dx}$: $y = x^2 e^{-x}$

Guidelines for Implicit Differentiation

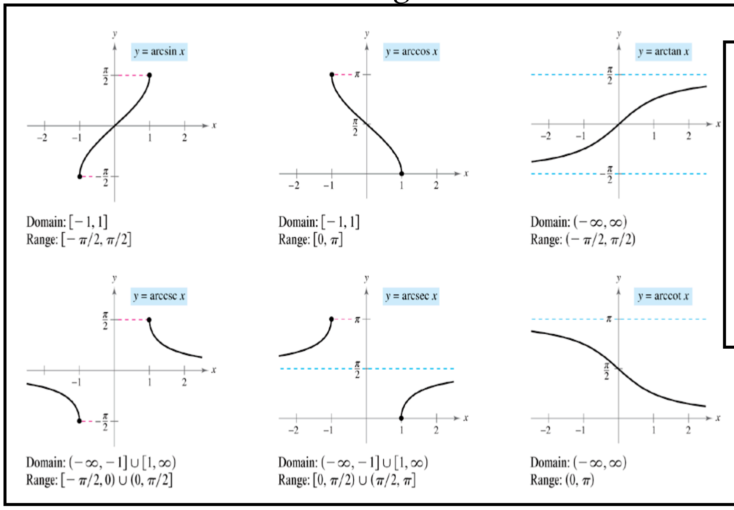
1. Differentiate both sides of the equation *with respect to x*.
2. Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
3. Factor dy/dx out of the left side of the equation.
4. Solve for dy/dx.

Ex.2 Find $\dfrac{dy}{dx}$: $e^{xy} + x^2 - y^2 = 10$

Ex.3 Integrate: $\int \frac{e^{\frac{1}{x^2}}}{x^3} dx$

Ex.4 Integrate: $\int \frac{e^{2x}}{1+e^{2x}} dx$

Section 5.6 *Inverse Trigonometric Functions: Differentiation*



THEOREM 5.16 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Ex.1 Find $f'(t)$: $f(t) = \arcsin\left(t^2\right)$

Ex.2 Find $h'(x)$: $h(x) = x^2 \arctan(x)$

Ex.3 Evaluate: $\sin\left(\arctan\left(\frac{3}{4}\right)\right)$

Ex.4 Write an algebraic form for $\sec\left(\arctan\left(4x\right)\right)$

Section 5.7 *Inverse Trigonometric Functions: Integration*

THEOREM 5.17 Integrals Involving Inverse Trigonometric Functions

Let u be a differentiable function of x, and let $a > 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

2. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Ex.1 Integrate: $\int \frac{t}{t^4+16} dt$

Ex.2 Integrate: $\int \frac{1}{x\sqrt{x^4-4}}dx$

Ex.3 Integrate: $\displaystyle\int \frac{2}{\sqrt{-x^2+4x}}dx$

Summary of Differentiation Rules

General Differentiation Rules

Let f, g, and u be differentiable functions of x.

Constant Multiple Rule:

$$\frac{d}{dx}[cf] = cf'$$

Sum or Difference Rule:

$$\frac{d}{dx}[f \pm g] = f' \pm g'$$

Product Rule:

$$\frac{d}{dx}[fg] = fg' + gf'$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$$

Derivatives of Algebraic Functions

Constant Rule:

$$\frac{d}{dx}[c] = 0$$

(Simple) Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}, \quad \frac{d}{dx}[x] = 1$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

Chain Rule

Chain Rule:

$$\frac{d}{dx}[f(u)] = f'(u)\,u'$$

General Power Rule:

$$\frac{d}{dx}[u^n] = nu^{n-1}\,u'$$

Basic Differentiation Rules for Elementary Functions

1. $\frac{d}{dx}[cu] = cu'$

2. $\frac{d}{dx}[u \pm v] = u' \pm v'$

3. $\frac{d}{dx}[uv] = uv' + vu'$

4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$

5. $\frac{d}{dx}[c] = 0$

6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$

7. $\frac{d}{dx}[x] = 1$

8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$

9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$

10. $\frac{d}{dx}[e^u] = e^u\,u'$

11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$

12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$

13. $\frac{d}{dx}[\sin u] = (\cos u)u'$

14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$

15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$

16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$

17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$

18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$

19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$

20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$

21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$

22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1 + u^2}$

23. $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$

24. $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

Basic Integration Rules ($a > 0$)

1. $\int kf(u)\,du = k\int f(u)\,du$

2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$

3. $\int du = u + C$

4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

5. $\int \frac{du}{u} = \ln|u| + C$

6. $\int e^u\,du = e^u + C$

7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$

8. $\int \sin u\,du = -\cos u + C$

9. $\int \cos u\,du = \sin u + C$

10. $\int \tan u\,du = -\ln|\cos u| + C$

11. $\int \cot u\,du = \ln|\sin u| + C$

12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$

14. $\int \sec^2 u\,du = \tan u + C$

15. $\int \csc^2 u\,du = -\cot u + C$

16. $\int \sec u \tan u\,du = \sec u + C$

17. $\int \csc u \cot u\,du = -\csc u + C$

18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$